Fisher’s Exact Test
Index

1. Introduction
   • How did Fisher derive the test?
   • What is Fisher’s exact test?
2. Computing in R…
   • Fisher’s exact test
   • What would happen if we had used the Chi-square test?
1920 → The Lady testing tea!

“(…) One of the women (B. Muriel Roach) was insisting that tea tasted different depending upon whether the tea was poured into the milk or whether the milk was poured into the tea. A short man (R. Fisher) pounced on the problem: *let us test the proposition!*”

How did Fisher derive the f-test? Let’s make a physical demonstration!

<table>
<thead>
<tr>
<th>Test</th>
<th>Red</th>
<th>Green</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Square</td>
<td>6</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Totals</td>
<td>8</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

- Is there **no association** between color and shape? Would any association turn as a result of nothing more than mere chance coincidence? → Lets call this “null hypothesis” (H0).
- Therefore the alternative would dictate shape and color as dependent variables → lets call this “alternative hypothesis” (H1).

*Salzburg, D.; The Lady Testing tea, how statistics revolutionized science in the twentieth century (1931)*
To generate a significance level we should consider only the cases where the marginal totals are the same, and among those, only the cases where the arrangement is at least as extreme as the observed arrangement:

With corresponding probabilities:

One-tailed probability: 
\[ .326 + .093 + .007 = .426 \]

Two-tailed probability: 
\[ .326 + .093 + .007 + .163 + .019 = .608 \]

R: The null hypothesis (H0) cannot be rejected at a 95% significance level

http://www.people.ku.edu/~preacher/fisher/fisher.htm
The Fisher exact test is a statistical significance test, used to determine if there are nonrandom associations between two categorical variables. It is generally applied under the following conditions:

i.  **The size of the sample is small**;

ii.  **The expected values in the cells of the contingency table are below 5**.

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>a</td>
<td>b</td>
<td>a+b</td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>c</td>
<td>d</td>
<td>c+d</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a+c</td>
<td>b+d</td>
<td></td>
<td>n</td>
</tr>
</tbody>
</table>

**Introduction**

What is the Fisher’s exact test? (1/2)

The probability of obtaining any such set of values is given by a **hypergeometric distribution**.


The probability of obtaining any such set of values is given by a hypergeometric distribution:

\[ p = \frac{(a+b)(c+d)}{n} \mathbin{/} \binom{a+c}{a} \binom{b+d}{c} = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{a!b!c!d!n!} \]

In probability theory and statistics, the hypergeometric distribution is a discrete probability distribution that describes the number of successes in a sequence of \( n \) draws from a finite population without replacement.

Assumptions:

- Fixed marginal totals
- Independent Row and column classifications (H0)


http://mathworld.wolfram.com/HypergeometricDistribution.html
Computing it in R…
Fisher’s exact test

```
# Fisher 2x2 Contingency table

# User input
values <- c(8,3,4,9)
names <- list(c("Triangle", "Square"), c("Red", "Green"))
alfa <- 0.05

# Contingency table
table <- matrix(values, nrow=2, dimnames = names, byrow=T)

# Fisher exact test
fisher.test(table, alternative = "t", conf.level = 1 - alfa) # "t" stands for two-tailed test

# Pearson Chi-square test
chisq.test(table, correct=F)
```

Fisher's Exact Test for Count Data

data: table
p-value = 0.09953
alternative hypothesis: true odds ratio is not equal to 1
95 percent confidence interval:
0.7624285 51.9058650
sample estimates:
odds ratio
5.512388

---

Monte Carlo

fisher.test(x, y = NULL, workspace = 200000, hybrid = FALSE,
control = list(), or = 1, alternative = "two.sided",
conf.int = TRUE, conf.level = 0.95,
simulate.p.value = TRUE, B = 2000)

---

Pearson's Chi-squared test

data: table
X-squared = 4.1958, df = 1, p-value = 0.04052