Comparing multiple groups and the power of a test

Uncertainty about reality

Probability as a numeric measurement of uncertainty about an event

Interval of confidence of a measurement based on a sample containing the true value with a given probability

(random sampling in a normal distribution t-distribution and mean bounded between confidence interval using mean and SE of sample. that can be defined at any level one chooses i.e. at any level of uncertainty)

Propagate the uncertainty

Statistical testing by defining a statistics (e.g. t, z, x², or, etc.) with known (or derivable) distribution such that we can bound it, at given significance level α

Traveling to a different time zone causes jet lag. Adjustment to the schedule of light resets the internal circadian clock. This change in the internal clock is called “phase shift”. Campbell and Murphy (1998) reported that human circadian rhythm could be reset by exposition of the back of the knee to light. The surprising result was regarded with skepticism by the community. The following data from Wright and Caesler (2002) reexamined this question.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Data (h)</th>
<th>Mean</th>
<th>n</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>-5.41, 1.13, 1.24, 2.53, -1.04, 2.49, 2.79</td>
<td>-0.3</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Knees</td>
<td>-3.7, 0.46, -1.58, 1.41, 4.2, 8.22</td>
<td>-0.3357</td>
<td>0.7908</td>
<td></td>
</tr>
<tr>
<td>Eyes</td>
<td>-0.78, -0.86, -1.35, -1.49, -1.52, -2.04, -2.83</td>
<td>-1.5514</td>
<td>0.76063</td>
<td></td>
</tr>
</tbody>
</table>

Retrieved from Whitlock & Schluter. The analysis of biological data. Roberts and Company publishers
Statistical test

\[ H_0: \mu_1 = \mu_2 = \mu_3 \]
\[ H_1: \text{at least one } \mu_i \text{ is different from the others} \]

What is the best statistics model to deal with this?

- Multiple pairwise comparisons with t-test
- Bonferroni correction of p-value
The principles of ANOVA:

\[ H_0: \mu_1 = \mu_2 = \mu_3 \]
\[ H_1: \text{at least one } \mu_i \text{ is different from the others} \]

Error mean square of ANOVA is the pooled sample variance, a measure of the variability within the groups.

Group mean square of ANOVA represents the variation among individuals belonging to different groups. It should be similar to the error mean square if population means are equal.

\[ \text{Error mean square } (\text{MS}_{\text{error}}) \text{ of ANOVA is the pooled sample variance, a measure of the variability within the groups} \]

\[ \text{MS}_{\text{error}} = \frac{\sum (n_i - 1)}{N - k} \]

- \( n_i \): number of individuals
- \( n_i \): number of individuals in group \( i \)
- \( s_i \): standard deviation in each group
- \( k \): number of groups

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\[ MS_{\text{groups}} = \frac{\sum_{i=1}^{k} n_i (Y_i - \bar{Y})^2}{k - 1} \]

\[ \bar{Y} = \frac{\sum_{i=1}^{k} n_i Y_i}{N} \]

- \( n_i \): number of individuals in group \( i \)
- \( Y_i \): mean of individuals in group \( i \)
- \( Y \): total mean
- \( k \): number of groups
- \( N \): total individuals

ANOVA table

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>df</th>
<th>Mean Squares</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups</td>
<td>SS_groups</td>
<td>k - 1</td>
<td>MS_groups</td>
<td>MS_groups/MS_error</td>
</tr>
<tr>
<td>Error</td>
<td>SS_error</td>
<td>N - k</td>
<td>MS_error</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>SS_total</td>
<td>N - 1</td>
<td>SS_total</td>
<td></td>
</tr>
</tbody>
</table>

The ANOVA statistics is the variance ratio, \( F \), which under the null hypothesis should lie close to 1

\[ F = \frac{MS_{\text{groups}}}{MS_{\text{error}}} \]

What is the sampling distribution of \( F \)?

\( F \) is distributed according to the so-called \( F \) distribution with \( k - 1 \) degrees of freedom for the numerator and \( N - k \) degrees of freedom for the denominator.

As usual, using the appropriate quantiles of the \( F \) distribution we can bound \( F \) according to the null hypothesis.
Variance Explained

$R^2$ is used in ANOVA to summarize the contribution of group differences to the variability in the data

$$SS_{total} = SS_{groups} + SS_{errors}$$

$$R^2 = \frac{SS_{groups}}{SS_{total}}$$

The test will define whether we can trust the alternative hypothesis that at least one of the groups has a different mean.

If you want to detect which group it is you can do:

**Planned comparison**

**Unplanned comparison**

Planned comparison

Difference between the means of the two planned groups

$$\bar{Y}_i - \bar{Y}_j$$

The standard error of the difference between the means

$$SE = \sqrt{\frac{MS_{error}}{n} + \frac{1}{N - k}}$$

with $N-k$ degrees of freedom.

The difference between the means is $t$-distributed with $N-k$ degrees of freedom.
To get a grasp of the power of the planned comparison let’s compare it’s 95% confidence interval with that of the two sample t-test

Go to R and the Practice Room

Home Work: Read about Kruskal-Wallis test

1. Make pairwise comparisons with Wilcoxon-Mann-Whitney test
2. Make pairwise comparisons with t-test
3. Do ANOVA without using the built-in function of R
4. Do ANOVA with the built-in function of R