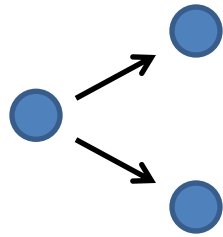


# Positive and negative feedbacks in populations

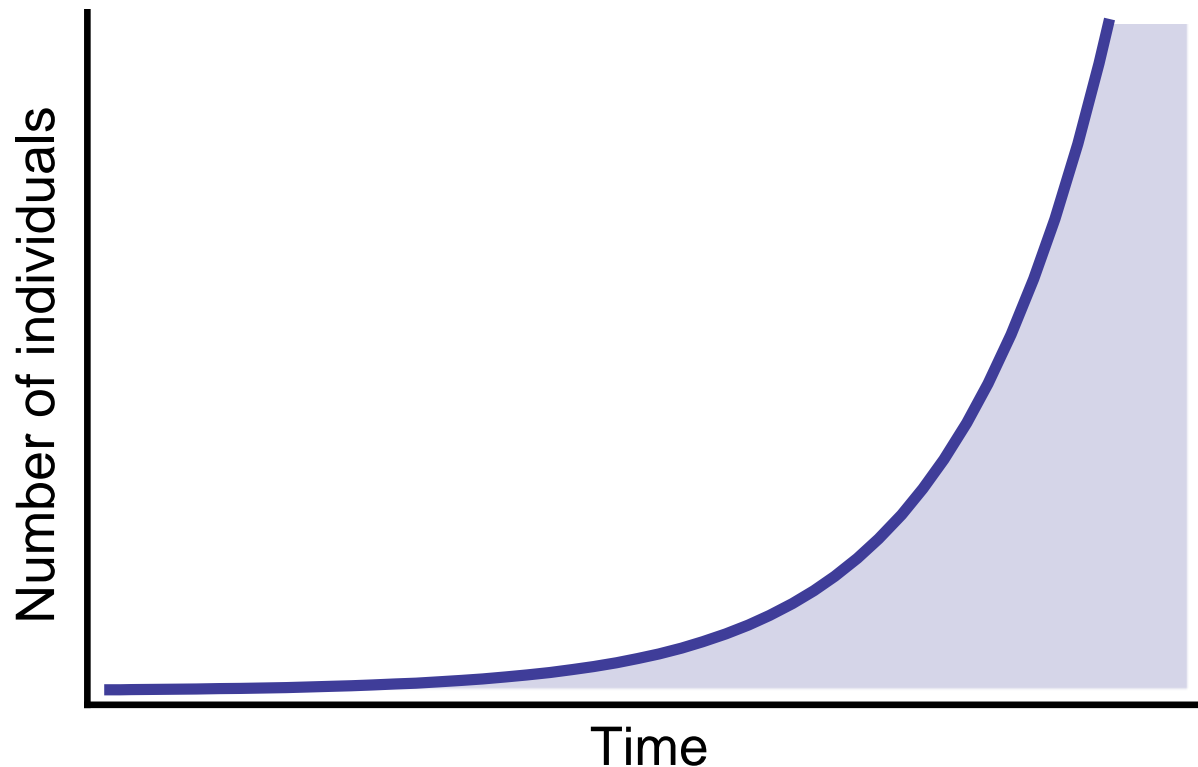
Ecology

# Positive feedback: Exponential growth



$$N_{t+1} = 2N_t$$

$$N_t = 2^t N_0$$



# How much time a bacterium needs to create a colony the size of the earth

- Mass of one bacterium:  $10^{-15}$  Kg
- Mass of the earth:  $10^{24}$  Kg
- Time for one generation: 0,5 hours

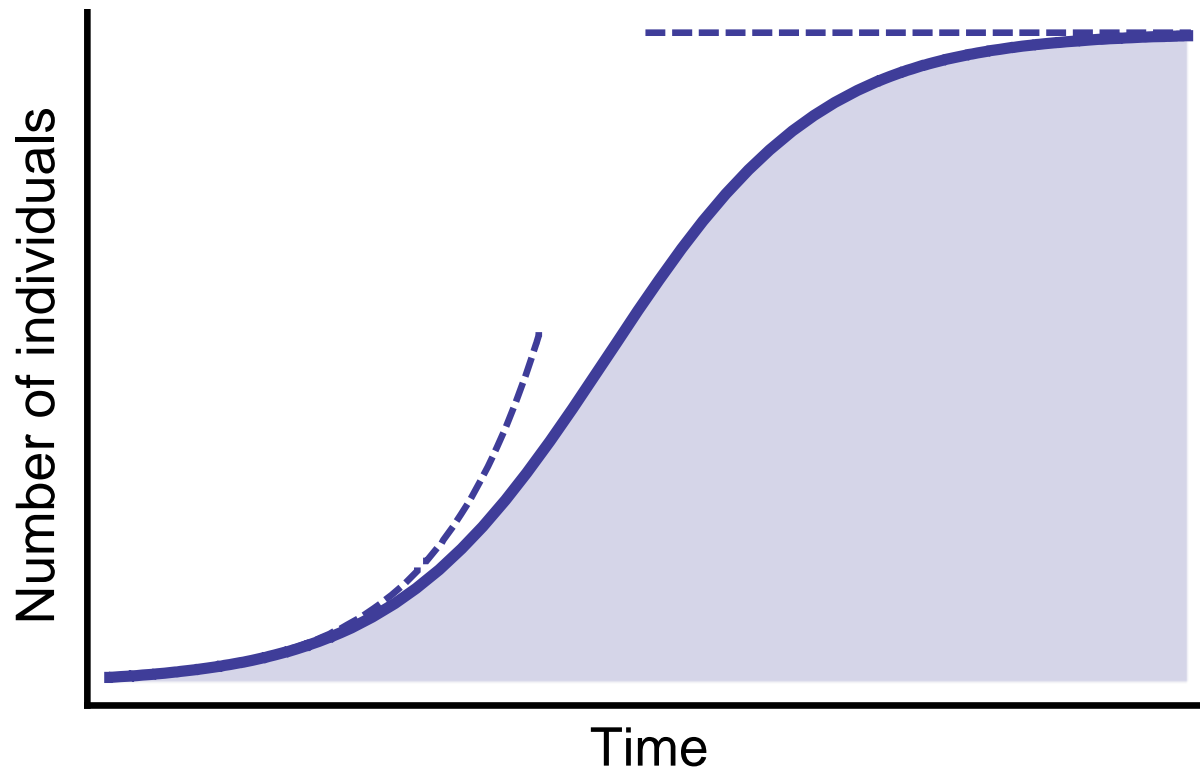
$$t = \log_2 \frac{N_t}{N_0}$$

# Negative feedback: density regulation

- Biotic factors:
  - Competition with other species (intra specific)
  - Competition within the same species
- Abiotic factors:
  - Nutrient depletion
  - Weather
  - ...

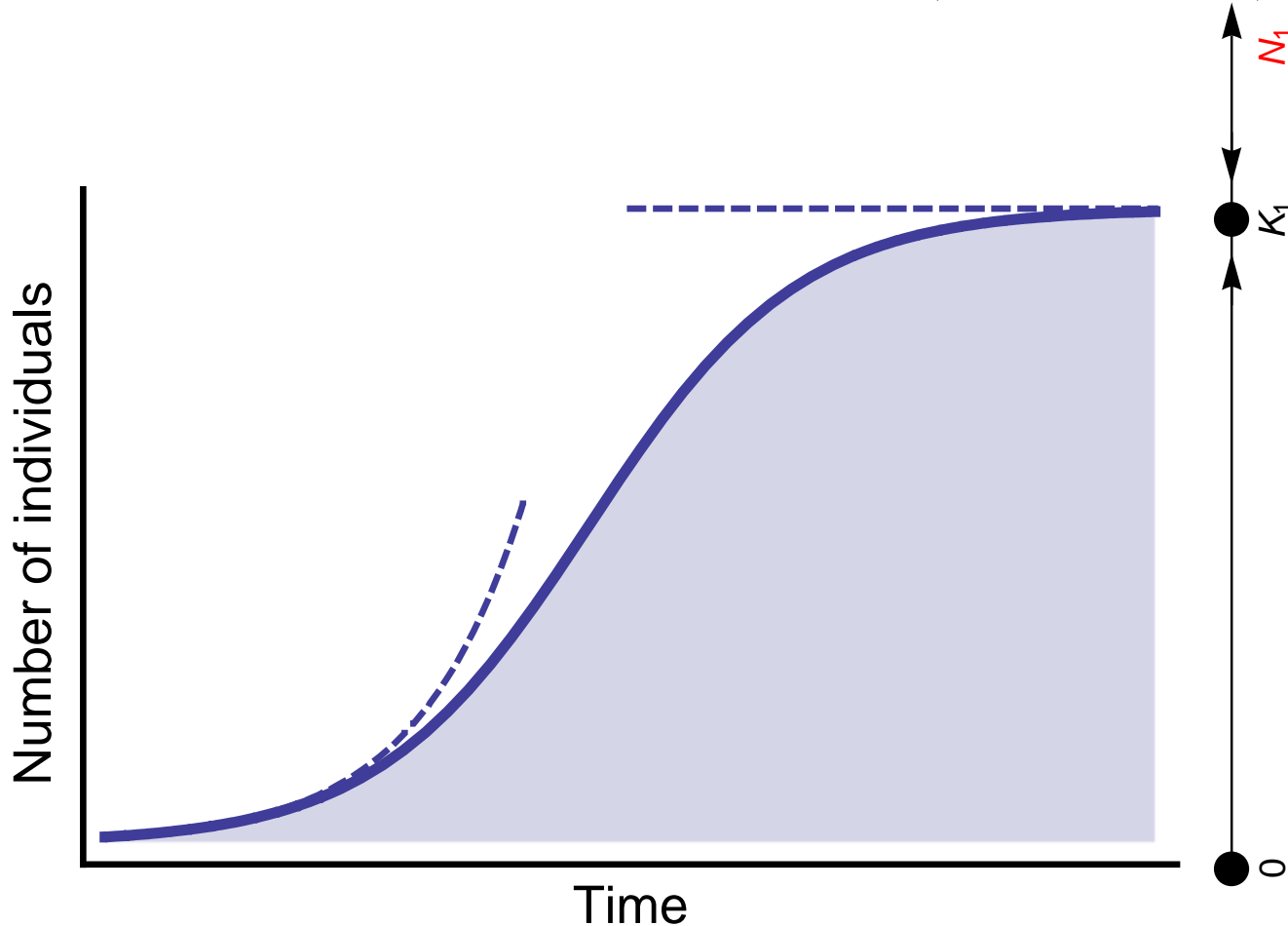
# Negative feedback: the logistic equation

$$N_{t+1} - N_t = \Delta N_t = r N_t (K - N_t)$$

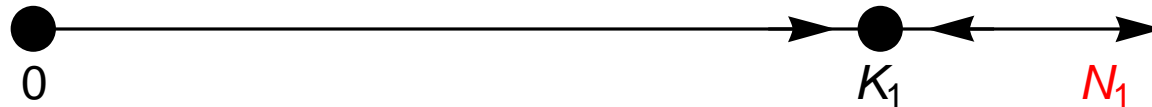


# Negative Feedback: the logistic equation

$$\Delta N_t = r N_t (K - N_t)$$



# Negative Feedback: the logistic equation



- Density regulation causes populations to stabilize
- This is typically because of limiting resources:
  - Space
  - Nutrients
  - Etc..
- What happens when a competitor comes in?

# Ecology:

## Interactions between species

$$\begin{aligned}\Delta N_1 &= r_1 N_1 (K_1 - N_1 + \alpha_{21} N_2) \\ \Delta N_2 &= r_2 N_2 (K_2 - N_2 + \alpha_{12} N_1)\end{aligned}$$

- Two species
- $\alpha_{12}$  and  $\alpha_{21}$  determine the nature of the interaction
  - $\alpha_{21} > 0$  : species 2 helps species 1 to grow
  - $\alpha_{21} < 0$  : species 2 prevents species 1 from growing

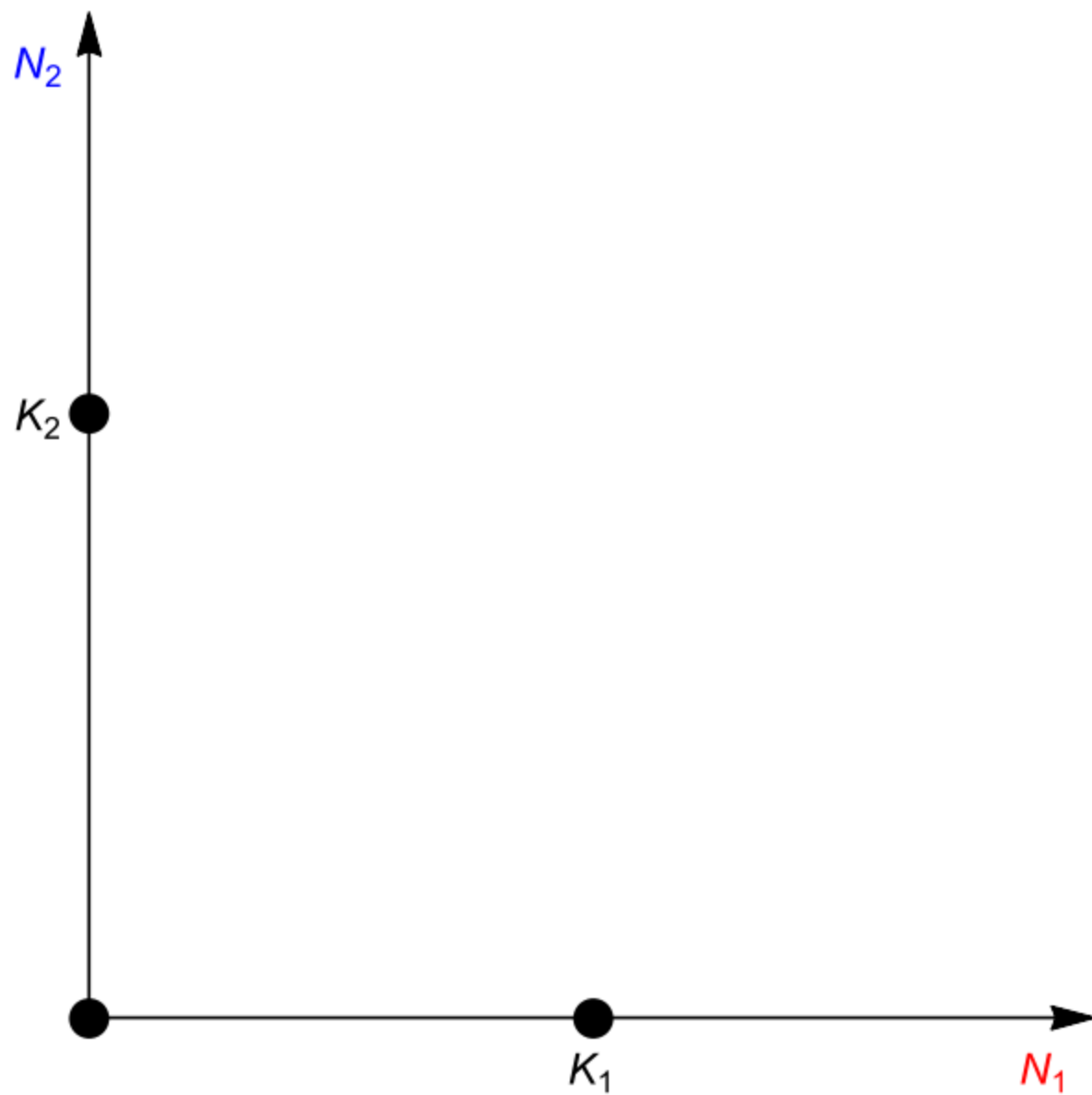


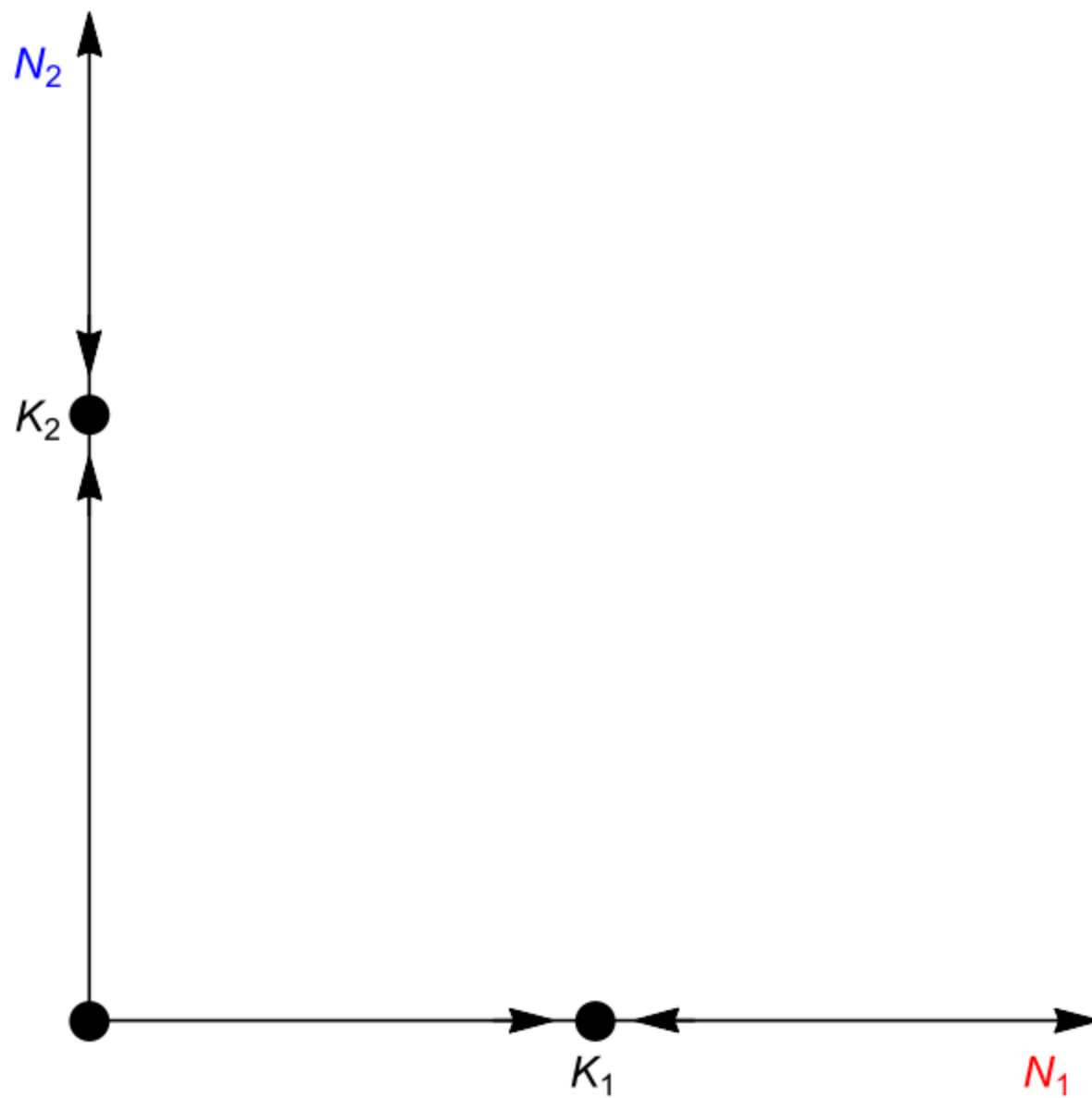
# Finding equilibria

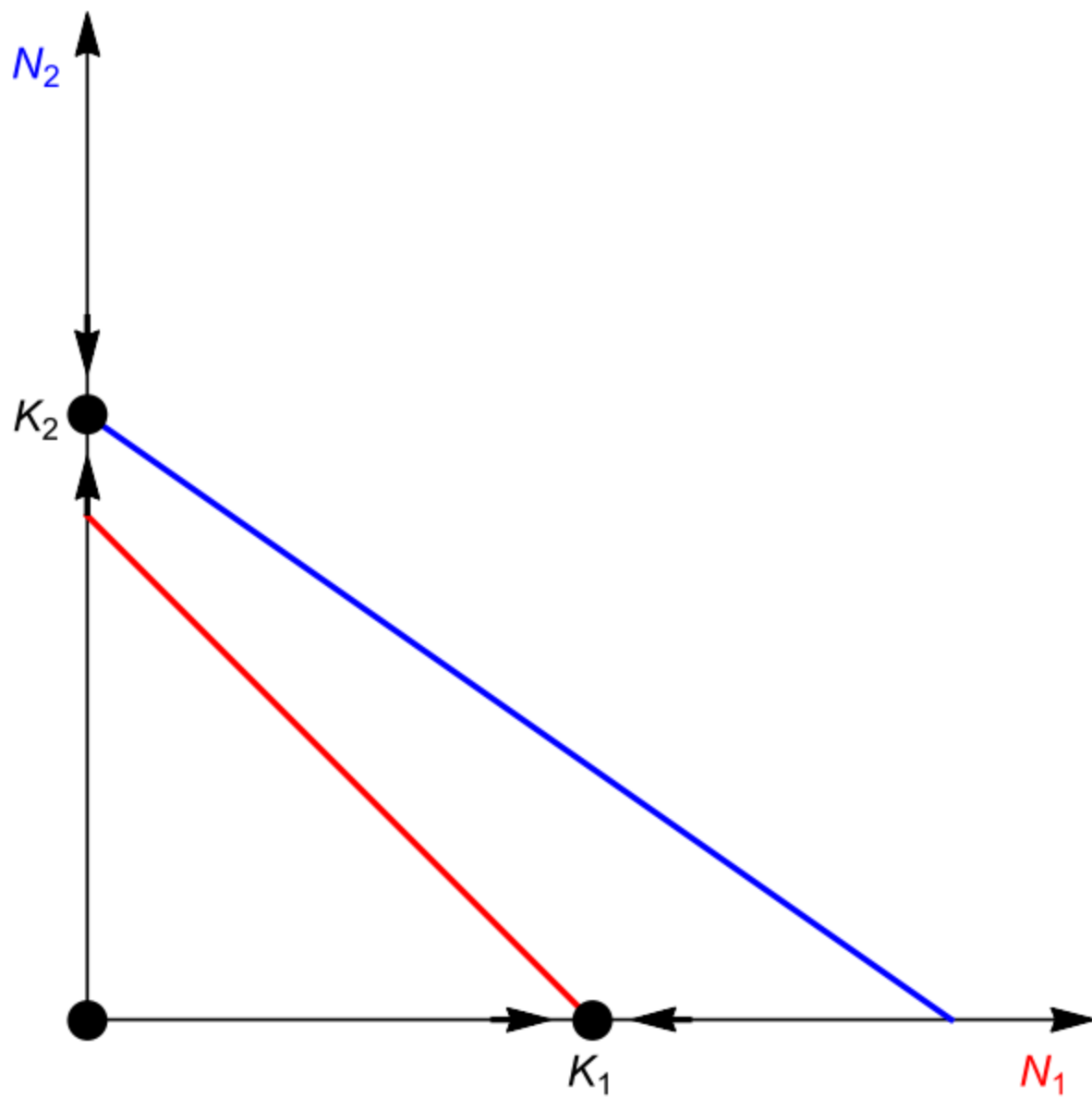
$$0 = r_1 N_1 (K_1 - N_1 + \alpha_{21} N_2)$$

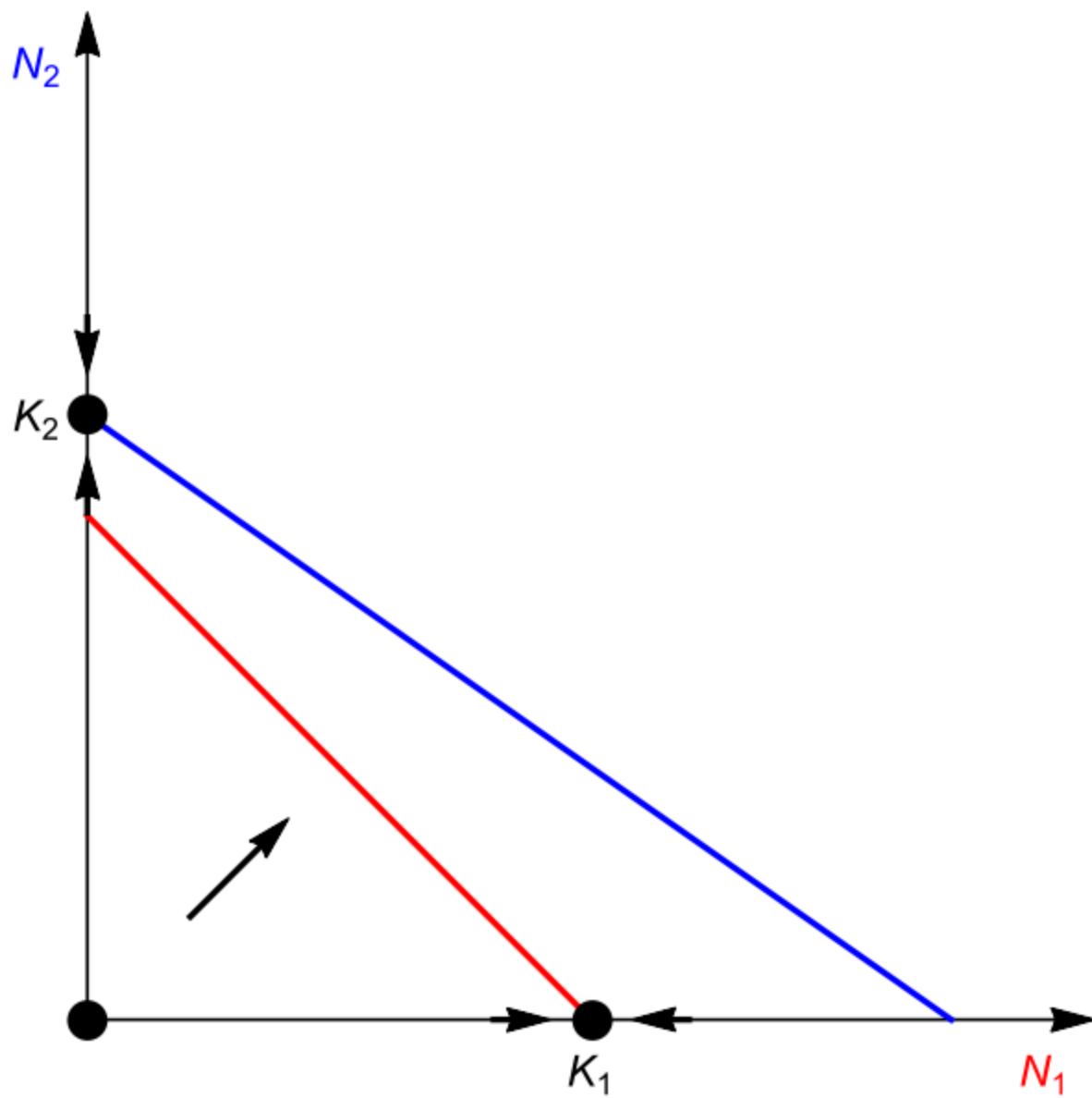
$$0 = r_2 N_2 (K_2 - N_2 + \alpha_{12} N_1)$$

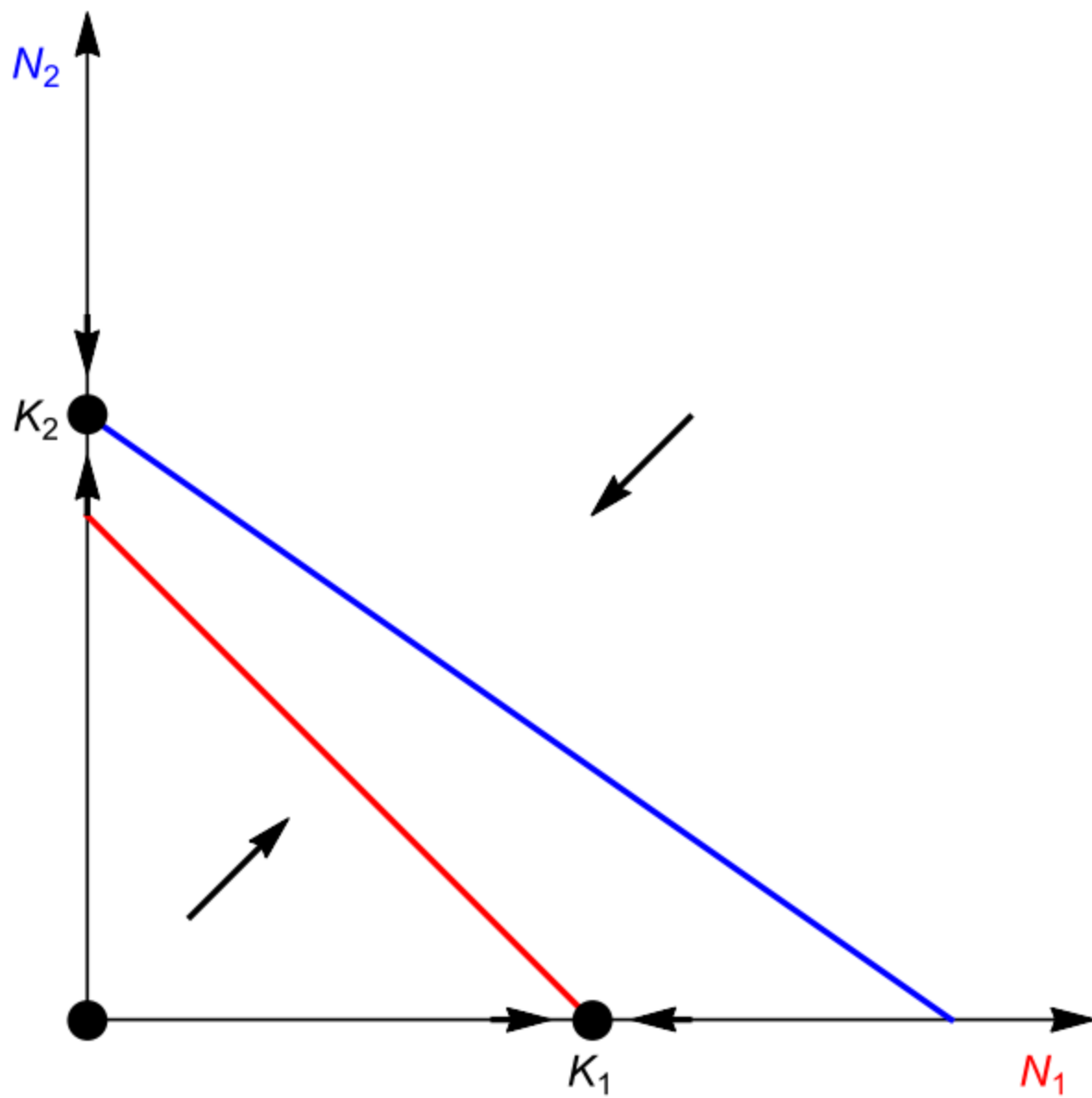
4 solutions:  $(0, 0)$   $(K_1, 0)$   $(0, K_2)$   $\left( \frac{K_1 + \alpha_{21} K_2}{1 - \alpha_{12} \alpha_{21}}, \frac{K_2 + \alpha_{12} K_1}{1 - \alpha_{12} \alpha_{21}} \right)$

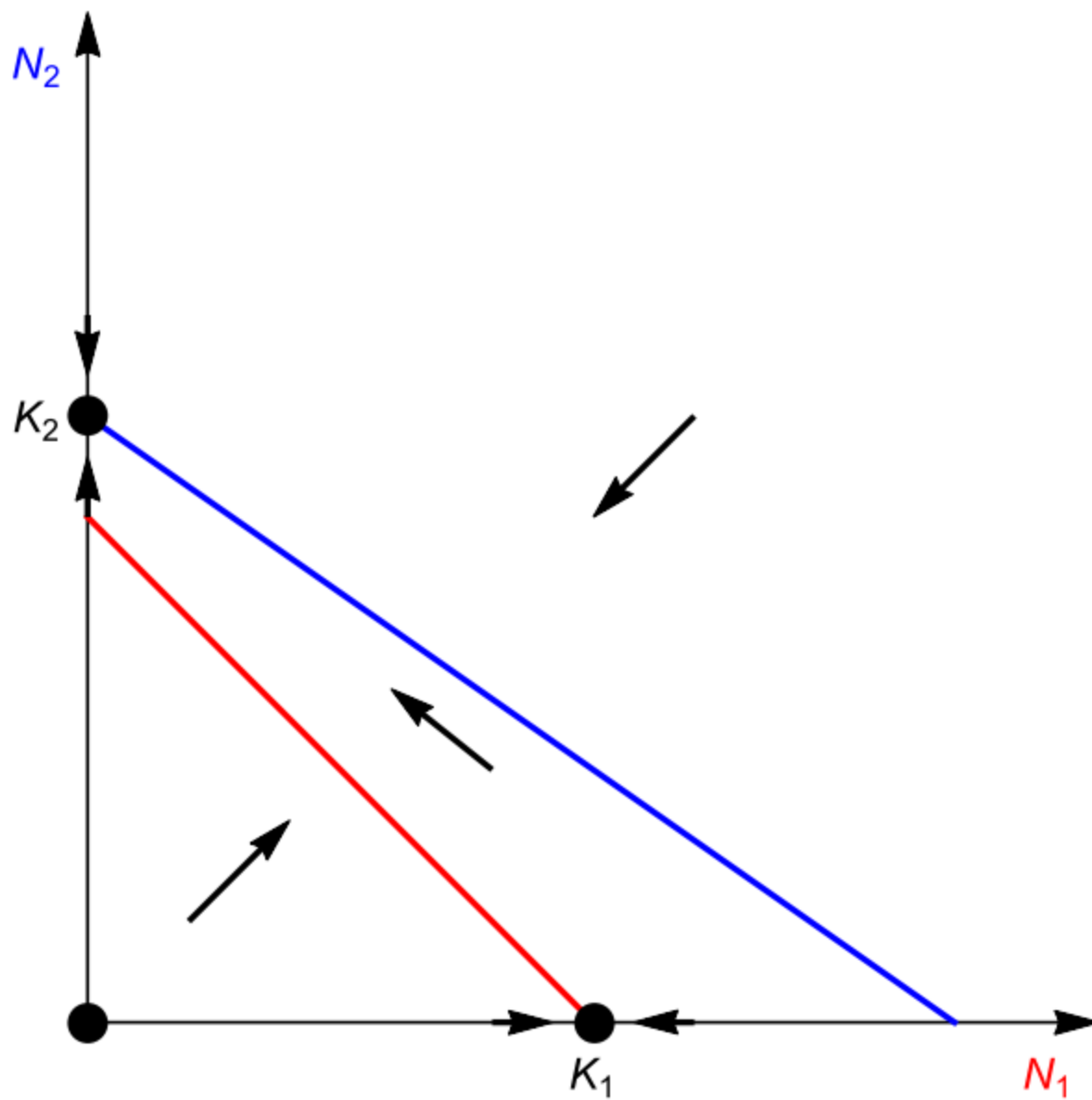




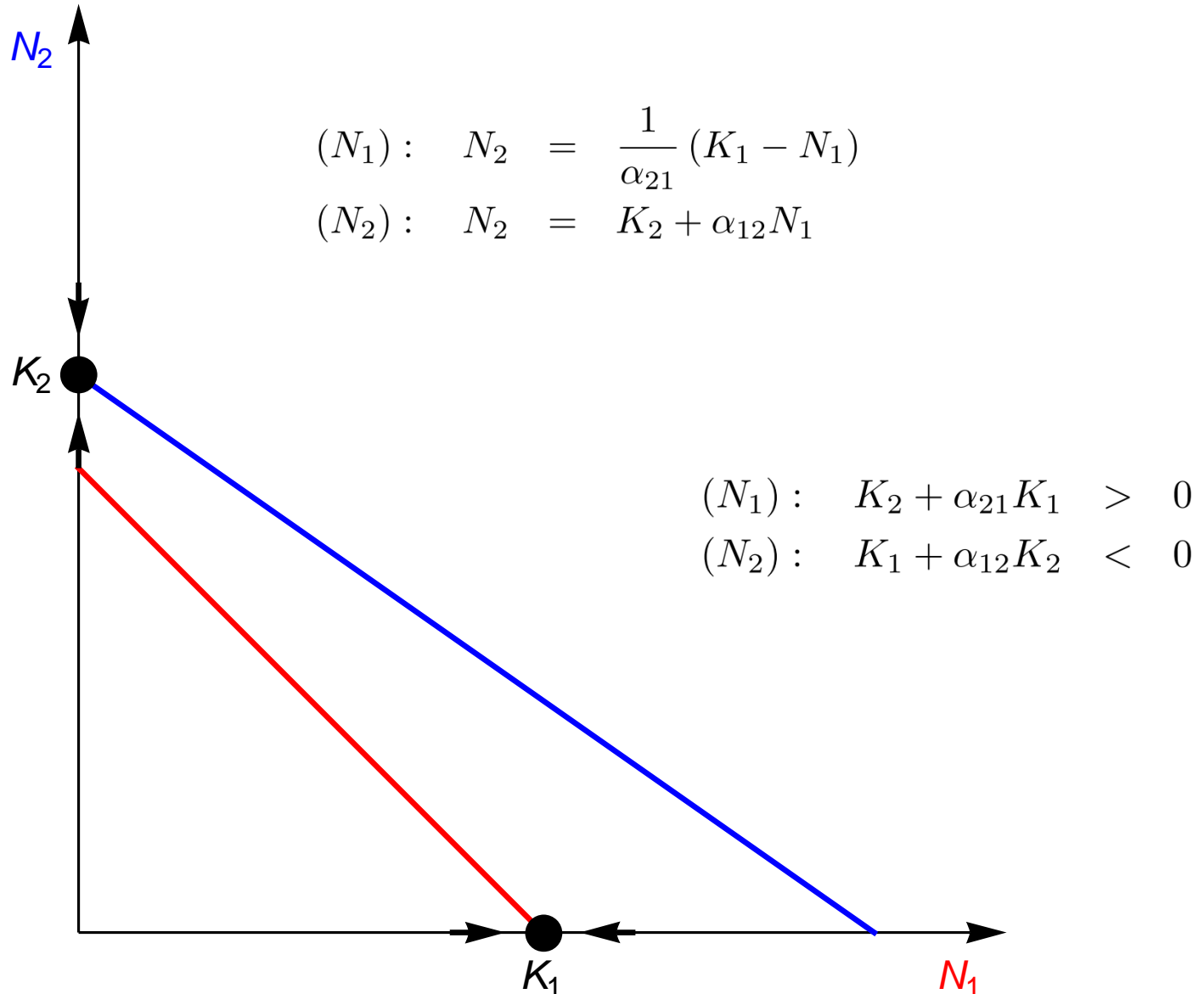






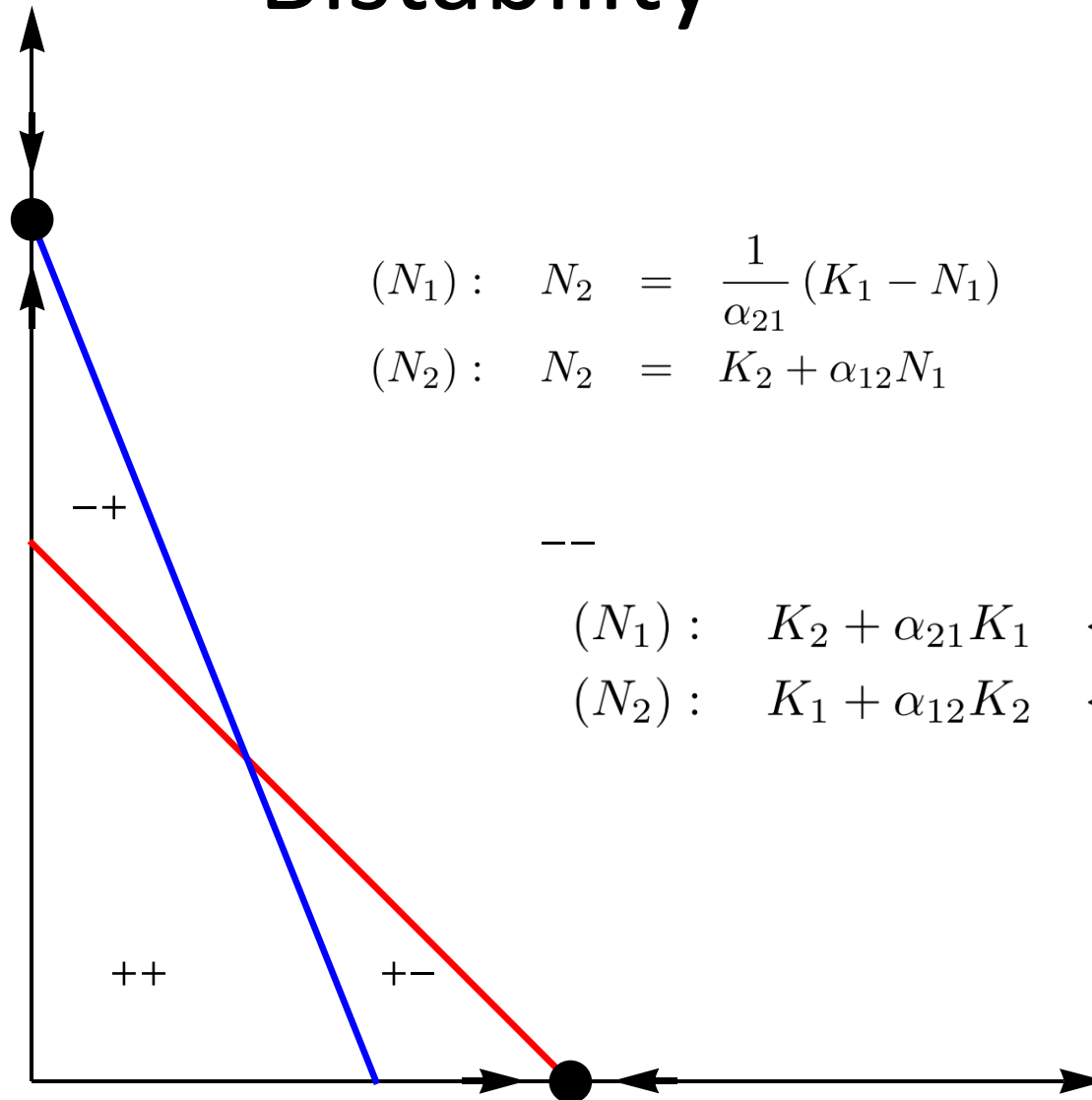


# Competitive Exclusion

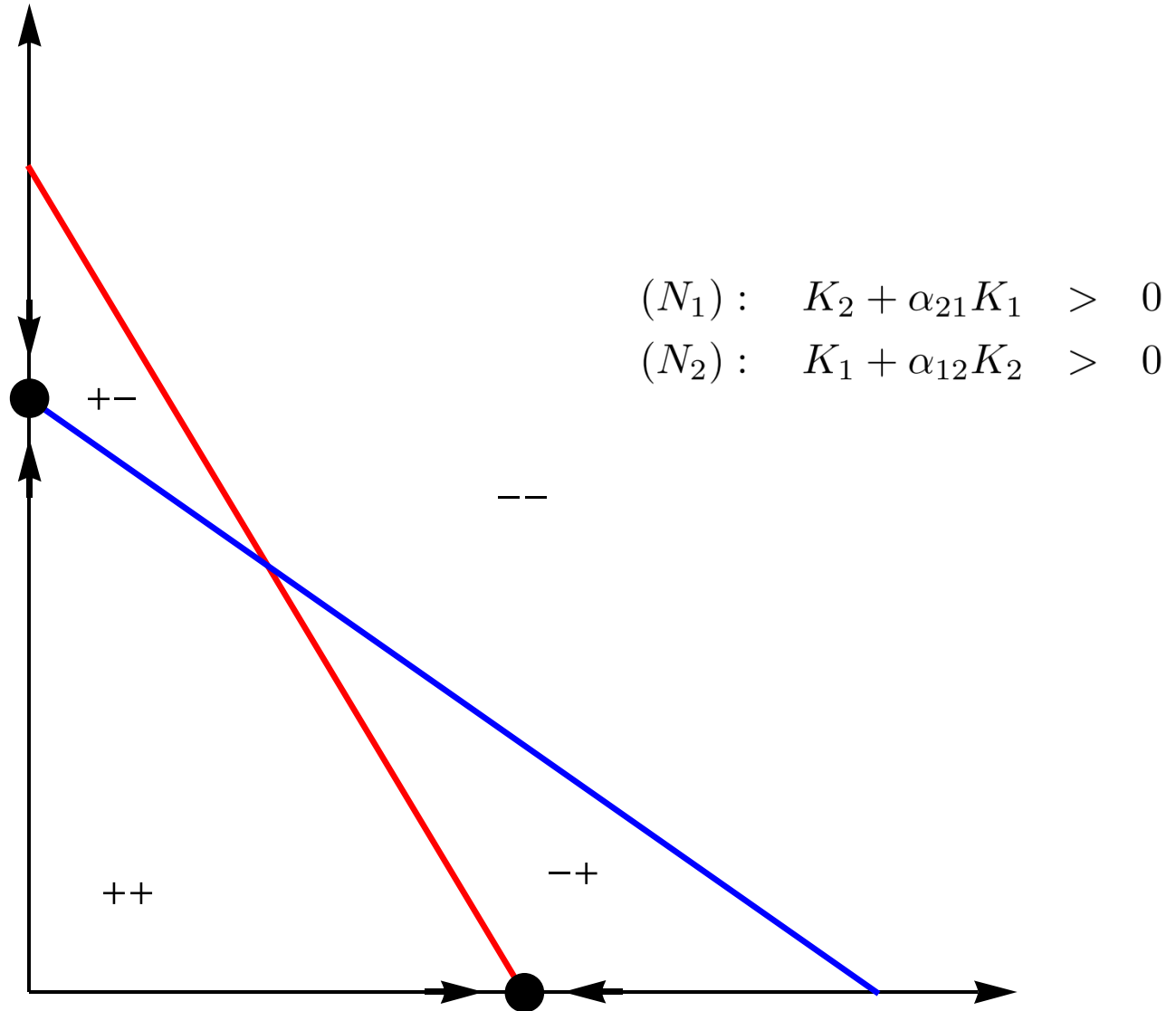




# Bistability



# Co-Existence



# Summary

- Populations are typically under density regulation
- This can be described mathematically
- Interactions between 2 species can have different outcomes, that depend on quantitative details
  - Carrying capacities and interaction between the species.