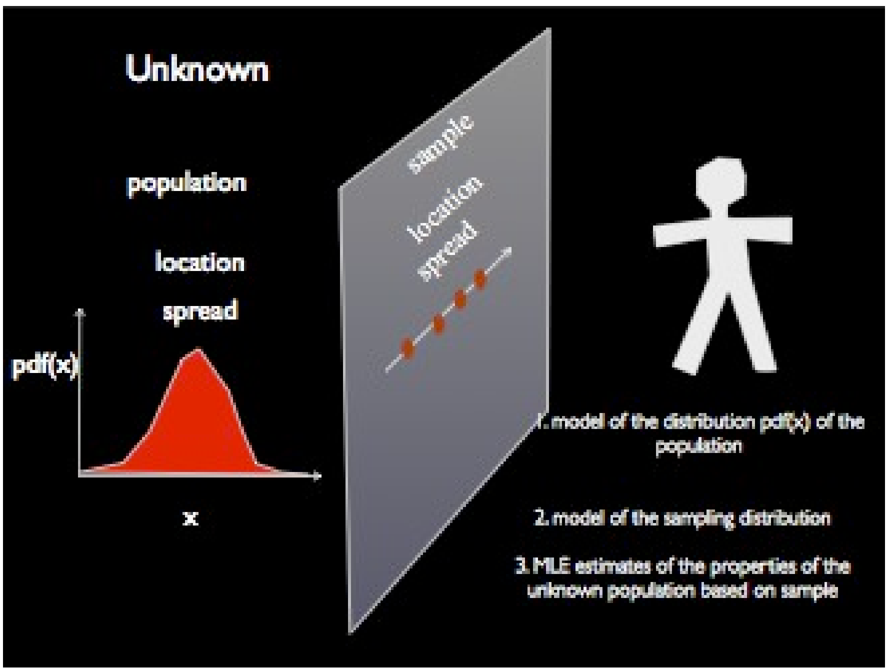
Thursday, November 20, 2014

# Hypothesis testing

# Recapitulating



Uncertainty about reality

Probability as a numeric measurement of uncertainty about an event

Maximum likelihood estimation of a population parameter based on a sample

(Interval of confidence of a measurement based on a sample)

Propagate the uncertainty

## Statistical hypothesis testing in biological research

**The scientific method**

1. Observation and description of a phenomenon or group of phenomena.
2. Formulation of a hypothesis to explain the phenomena. Often the hypothesis takes the form of a causal relationship or mechanism.
3. Use of the hypothesis to predict the existence of other phenomena, or to predict quantitatively the results of new observations.
4. Performance of experimental tests of the predictions by several independent experimenters and properly performed experiments.

## Statistical hypothesis testing in research

Occam's razor

*Numquam ponenda est pluritas sine necessitate*

*"Shave off" (omit) unnecessary entities in explanations*

Choose the most parsimonious interpretation

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Differences may and will occur simply by chance.

How can we determine in any given case whether the observed differences between two samples are due merely to chance or are caused by other factors?

Statistical inference procedures.

**Hypothesis Testing**

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## Hypothesis Testing

"Standard" hypothesis testing procedure

1. In face of a biological problem formulate a hypothesis and design an experiment with an expected yes-or-no result
2. Collect the data.
3. Do a panel of statistical tests
4. Find the one with the smallest p-value
5. Choose a level of significance that best suites your favorite interpretation of the data

6. In case of panic, call someone that you believe understands statistics

## Hypothesis Testing

### Inference procedure

1. State the null hypothesis ($H_0$) and its alternative ($H_1$). Decide what data to collect and under what conditions.

2. Choose a test, the model of which most closely approximates the conditions of the research in terms of the assumption on which the test is based.

3. Find the sampling distribution of the statistical test under the assumption that $H_0$ is true.

4. Specify a significance level ($\alpha$) and a sample size ($n$).

5. On the basis of 2, 3, and 4 above, define the region of rejection for the statistical test.

6. Collect the data. Using the data compute the value of the test statistic. If the value is in the region of rejection, the decision is to reject $H_0$; otherwise, the decision is that $H_0$ cannot be rejected at the chosen level of significance.

## Hypothesis Testing

The null hypothesis $H_0$

It is formulated with the purpose of being rejected

It is the negation of the point one is trying to make

It is an hypothesis of "no effect"

## Hypothesis Testing

The test statistic

How to choose a test statistic?

## Hypothesis Testing

### The level of significance

In advance of the data collection, we specify all possible samples that could occur when H0 is true.

From these we specify a subset of possible samples which are so inconsistent with H0 (so extreme) that the probability is very small, when H0 is true, that the sample we actually observe will be among them.

Then if we actually observe a sample which is included in that subset, we reject H0 in favor of H1.

We reject H0 in favor of H1 if a statistical test yields a value whose associated probability of occurrence under H0 is equal to or less than some small probability, usually denoted $\alpha$, called the level of significance.

## Hypothesis Testing

Find the sampling distribution of the test statistic under the null hypothesis H0

Find all possible samples under the null hypothesis H0

Compute the test statistic for each sample

Find the probability "associated with" each value of the test statistic

## Hypothesis Testing

The "infamous" p-value

p is the probability "associated with" a specified value of the test statistics

It is the probability of drawing a sample with a test statistic value <u>at least as extreme</u> as the specified value

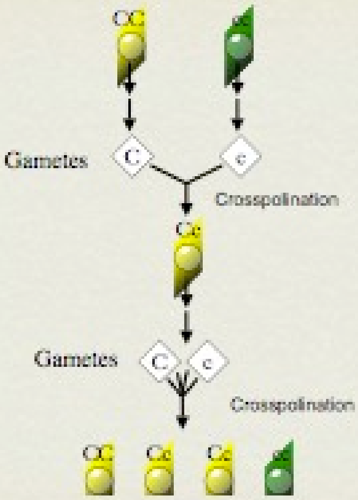
# Hypothesis Testing Example

Does a recessive mutation *pem* lead to early developmental problems ?

Does the mutation *pem* in homozygotic individuals increase the mortality in the embryo?

A suitable null hypothesis: *pem* shows Mendelian segregation

$H_0$: $\pi_{pem}$ =1/4 versus $H_1$: $\pi_{pem}$<1/4




| | c 1/1 |
|---|---|
| C 1/1 | Cc 1/1 |

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| C 1/2 | CC 1/4 | Cc 1/4 |
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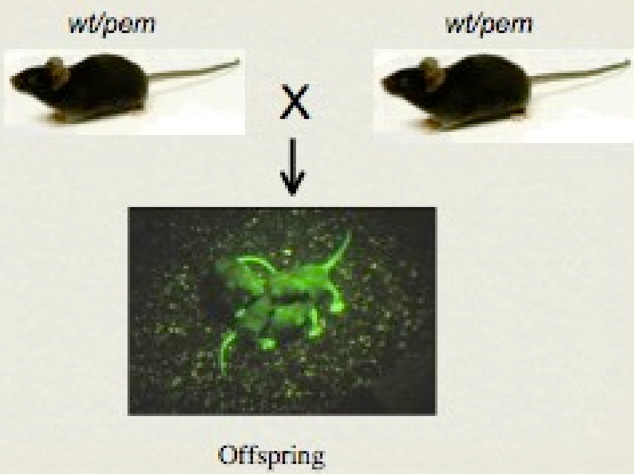
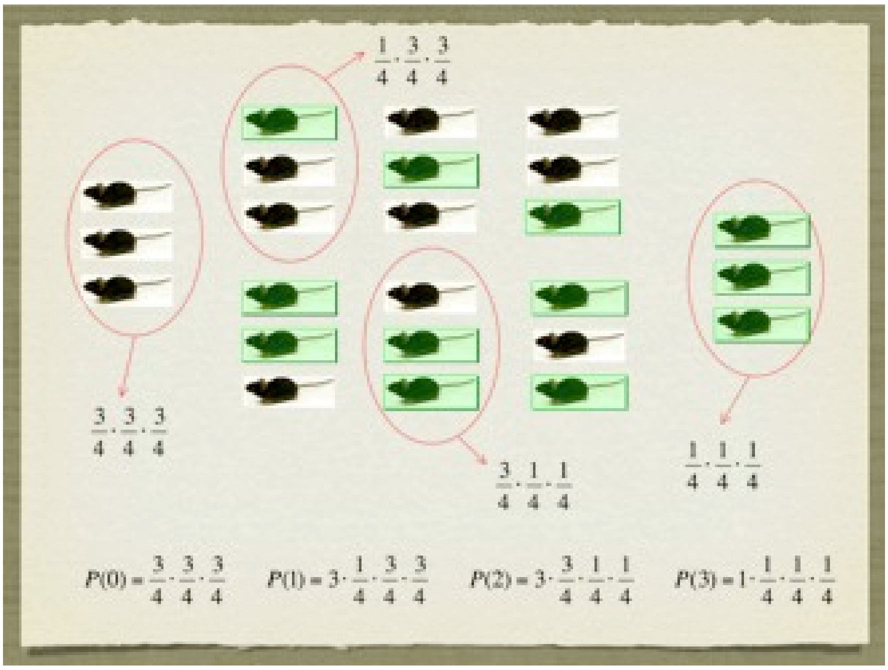
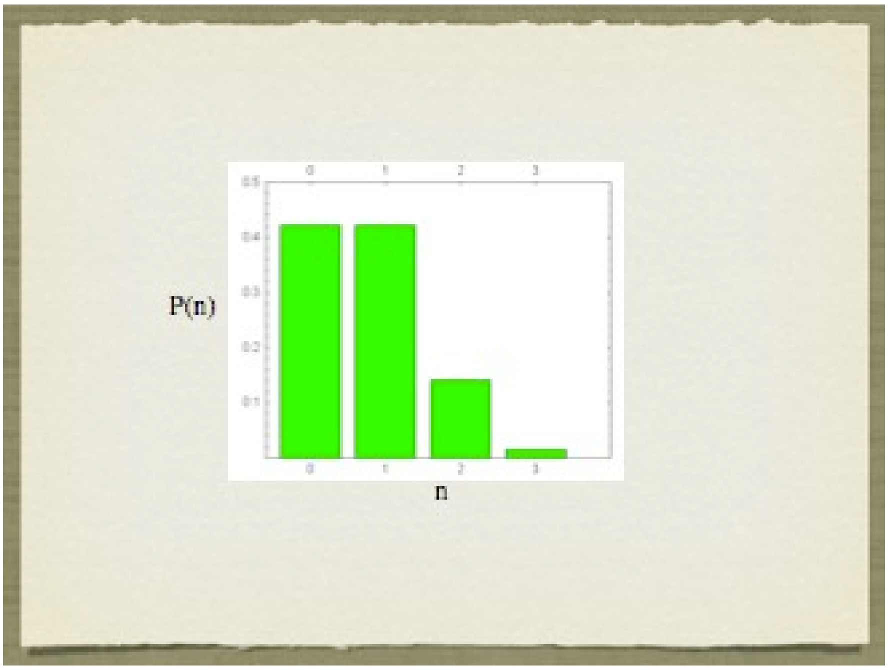
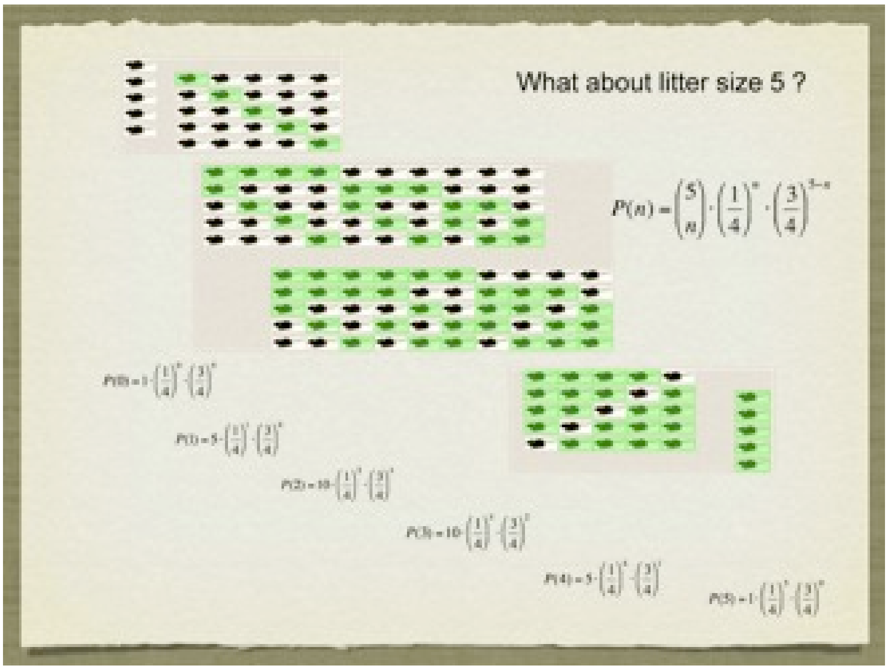




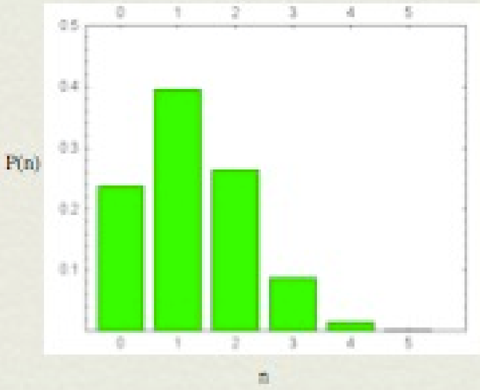
$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$
$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$
$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}$
$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}$
$P(0)=\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$
$P(1)=3\cdot\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$
$P(2)=3\cdot\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}$
$P(3)=1\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}$

P(n)
n

What about litter size 5 ?
$P(n)=\binom{5}{n}\cdot\left(\frac{1}{4}\right)^{n}\cdot\left(\frac{3}{4}\right)^{5-n}$

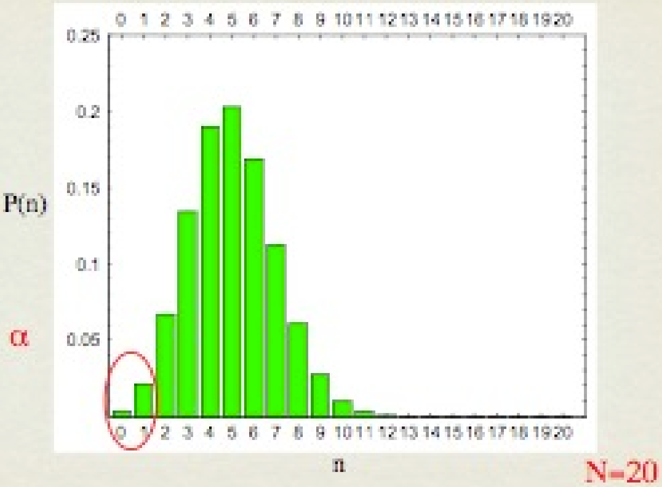


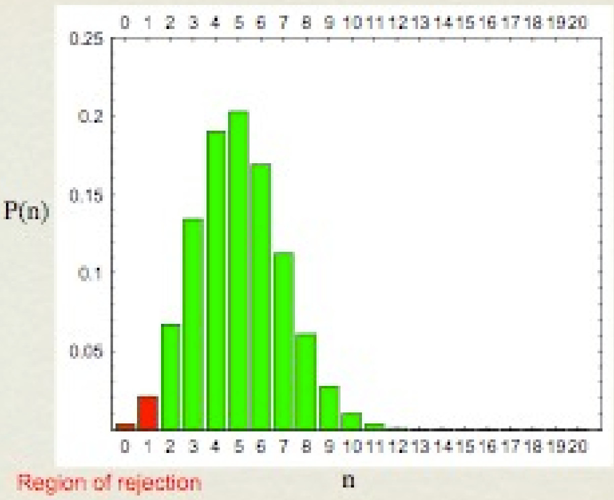

What about litter size 9 ?

Binomial Distribution

$$P(n) = P_{\pi}(n \mid N) = \binom{N}{n} \cdot \pi^{n} \cdot (1-\pi)^{N-n}$$

$$= \frac{N!}{n!(N-n)!} \cdot \pi^{n} \cdot (1-\pi)^{N-n}$$




Collect data

If *n* (the number of *pem* homozygous) is equal or smaller to 1 then reject the null hypothesis that pem has Mendelian segregation

Otherwise, there is no evidence for non-Mendelian segretion

## Hypothesis Testing

### Types of errors

Type I error involves rejecting the hypothesis $H_0$ when it is, in fact, true.

Type II error involves failing to reject the null hypothesis when, in fact, it is false.

Note that $\alpha$ gives the probability of mistakenly or falsely rejecting $H_0$.

The probability of errors of type II, denoted $\beta$, is more difficult to specify and one generally specifies *N* the sample size and $\alpha$, from which $\beta$ can be derived.

## Hypothesis Testing

### Error Probabilities

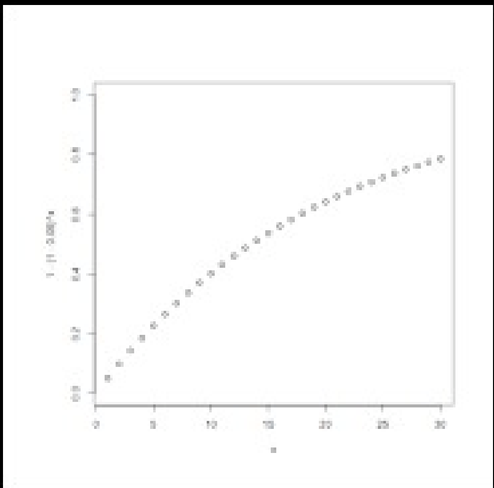
| | Accept $H_0$ | Reject $H_0$ |
|---|---|---|
| $H_0$ True | $1-\alpha$ | $\alpha$ |
| $H_1$ True | $\beta$ | $1-\beta$ |

## Hypothesis Testing

One-tailed and two-tailed

Importance of Hypothesis $H_1$

# The issue of multiple testing

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